

COMMENTS ON ENVIRONMENTAL EFFECTS IN THE ORIGIN OF ANGULAR MOMENTA IN GALAXIES

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ABSTRACT. We examine the orientations of galaxies in 43 rich Abell galaxy clusters belonging to superclusters and containing at least 100 members in the considered area as a function of supercluster multiplicity. It is found that the orientation of galaxies in the analyzed clusters is not random and the alignment decreases with supercluster richness, although the effect is statistically significant only for azimuthal angles. The dependence of galaxy alignment on cluster location inside or outside a supercluster and on supercluster multiplicity clearly shows the importance of environmental effects on the origin of galaxy angular momenta. The comparison with alignment of galaxies in a sample of rich Abell clusters not belonging to superclusters is made too.

Key words: galaxies, angular momenta.

1. Introduction

One of the most important but unsolved until now problems in modern extragalactic astronomy and cosmology is the origin of large scale structures. At present the Λ CDM model is commonly accepted as the basis by which cosmic structures were born. In the model the Universe is considered to be spatially flat, homogeneous and isotropic at an appropriate scale. However, the dimension of that scale is changing with the growth in our knowledge of the Universe. In addition, it is also commonly accepted that currently observed structures originated from nearly isotropic distributions in the early Universe. The departure from isotropy, as estimated by the CMBR is on the order of $\delta\rho/\rho \simeq 10^{-5}$. About half a century ago the main problems were connected with the types of perturba-

tions, their amplitude, and scale (mass or length). In the Λ CDM model the structures were formed in the primordial, adiabatic, nearly scale invariant, Gaussian, random fluctuations.

Numerous different theories of galaxy origins predict various means by which galaxies gained angular momentum (Peebles 1969, Zeldovich 1970, Sunyaev & Zeldovich 1972, Doroshkevich 1973, Shandarin 1974, Wesson 1982, Silk & Efstathiou 1983, Bower et al. 2005). Since different scenarios forecast different distributions for the angular momenta of galaxies in structures (Peebles 1969, Doroshkevich 1973, Shandarin 1974, Silk & Efstathiou 1983, Catelan & Theuns 1996, Li 1998, Lee & Pen 2002, Navarro et al. 2004, Trujillo et al. 2006), testing galaxy orientations can be used to check the scenarios of galaxy origins. Normally studies of the orientation of galaxy planes were performed.

Godłowski & Flin (2010) studied the orientation of galaxy groups in the Local Supercluster, and found a strong alignment of the major axis of the groups with directions towards the supercluster center (Virgo cluster) as well as with the line joining the two brightest galaxies in the group. The interpretation of these observational features is as follows. The brightest galaxies (believed to be the most massive ones) of the group originated first. As a result of gravitational forces, other galaxies were attracted to them and a filament was formed at the end.

Similar results were obtained by Paz et al. (2011), where the authors found a strong alignment between the projected major axis of group shapes and the surrounding distribution of galaxies to scales of $30h^{-1}Mpc$. Smargon et al. (2011) searched for two types of cluster alignments using pairs of clusters: the alignment between the projected major axes of

the clusters displayed a weak effect up to $20h^{-1}Mpc$, whereas the alignment between the major axis of one cluster with the line connecting the other cluster in the pair displayed a strong alignment on scales up to $100h^{-1}Mpc$. Also, a statistically significant anisotropy for the galaxy groups and cluster orientations for a sample of the Jagellonian field was noted by Flin & Vavilova (1996) and Vavilova (1999).

The other possibility for interpreting the (Godłowski & Flin, 2010) result is that the galaxies form at a pre-existing filament. Consistent with that argument are the results of Jones et al. (2010), who found that the spins of spiral galaxies located within cosmic web filaments tend to be aligned along the larger axis of the filament. Jones et al. (2010) interpreted it as “fossil” evidence, indicating that the action of large scale tidal torques effected the alignment of galaxies located in cosmic filaments. The relationship between alignment and the surrounding neighborhood was observed in a study of orientations in the vicinity of voids by Varela et al. (2011), a continuation of an earlier study of galaxy orientations in regions surrounding bubble-like voids (Trujillo et al., 2006). Varela et al. (2011) found that the observed tendency in the alignment of galaxies is similar to that observed in numerical simulations of the distribution of dark matter, i.e. in distributions of the minor axis of dark matter halos around cosmic voids, which suggests a possible link to the evolution of both components.

The large scale distribution is usually known as the “Cosmic Web.” In practice the “Cosmic Web” has four components which are: long filaments, walls, voids, and rich, dense regions – so called galaxy clusters. Thus, we should investigate the alignment of galaxies and clusters in such structures as well.

In Godłowski et al. (2010), Paper I hereinafter, a sample of 247 rich Abell clusters was analyzed. It was found that the alignment of members in rich structures containing more than 100 galaxies is a function of the group mass, in the sense that the alignment increases with the richness of the group. In view of such features, it is interesting to see if clusters belonging to the larger structures exhibit the same type of alignment as the entire sample of clusters. For that reason, we Godłowski et al. (2011, Paper II hereinafter) analyzed the alignment of galaxy cluster members for clusters belonging to superclusters. The problem was not investigated previously, although the alignment of galaxies in superclusters has been investigated many times.

In Paper II the alignment of galaxies in the sample of 43 rich Abell galaxy clusters belonging to a supercluster and having at least 100 members was investigated. It was found that the orientation of galaxies in the analyzed clusters is not random. However, significant differences were found with the results obtained in Paper I, in which an increase of alignment was found

for rich Abell clusters as a function of cluster richness. On the contrary, other clusters belonging to superclusters do not show such an effect. In Paper I galaxies in the sample studied were split into three bins according to supercluster multiplicity. They were: a subsample of superclusters containing only 4 structures, a subsample of superclusters containing 5–7 structures, and finally a subsample of superclusters containing 8–10 clusters. However, because the analysis was based on only 3 bins, it was difficult to determine the statistical significance of the results. In the present paper we decided to analyse the orientation of galaxies in clusters belonging to supercluster in more detail, without binning on clusters properties such as richness or BM type. In essence, we used the likelihood of membership in a supercluster as the parameter which characterizes each analyzed cluster.

2. Observational data

Input data for the present study made use of the PF Catalogue of galaxy structures (Panko & Flin, 2006). That Catalogue was constructed by finding structures in the Muenster Red Sky Survey (MRSS) (Ungruhe et al., 2003) in conjunction with the Voronoi tessellation technique applied to find structures. The MRSS is a large-sky galaxy catalogue covering an area of about 5000 square degrees in the southern hemisphere. It is the result of scanning 217 ESO plates, yielding positions, red magnitudes, radii, ellipticities, and position angles for about 5.5 million galaxies, and is complete to $r_F = 18.3^m$. As a result, there are 6188 galaxy structures called clusters. Structure ellipticities and position angles were determined by means of the standard covariance ellipse method. We have selected a sample of 247 very rich clusters containing at least 100 members each that are identified with an ACO cluster (Abell et al., 1989) – see Paper I for more details. Unfortunately there are not obvious correlation between “rich” PF clusters and Abell’ richness classes. The PF catalogue was also used as the basis for supercluster search (see, for example, Panko, 2011) and 54 superclusters containing at least 4 clusters each were detected. We found that 43 of a total of 247 rich PF clusters belong to superclusters, and they were chosen for detailed analysis. However, it should be noted that three clusters, 0347-5571, 2217-5177, and 2234-5249, have two possible identifications with superclusters of different multiplicity, so must be counted in two bins, which formally enlarged our sample to 46 clusters.

3. Results and Discussion

Studies of galaxy alignments are usually done by analyzing distributions of the angles connected with the

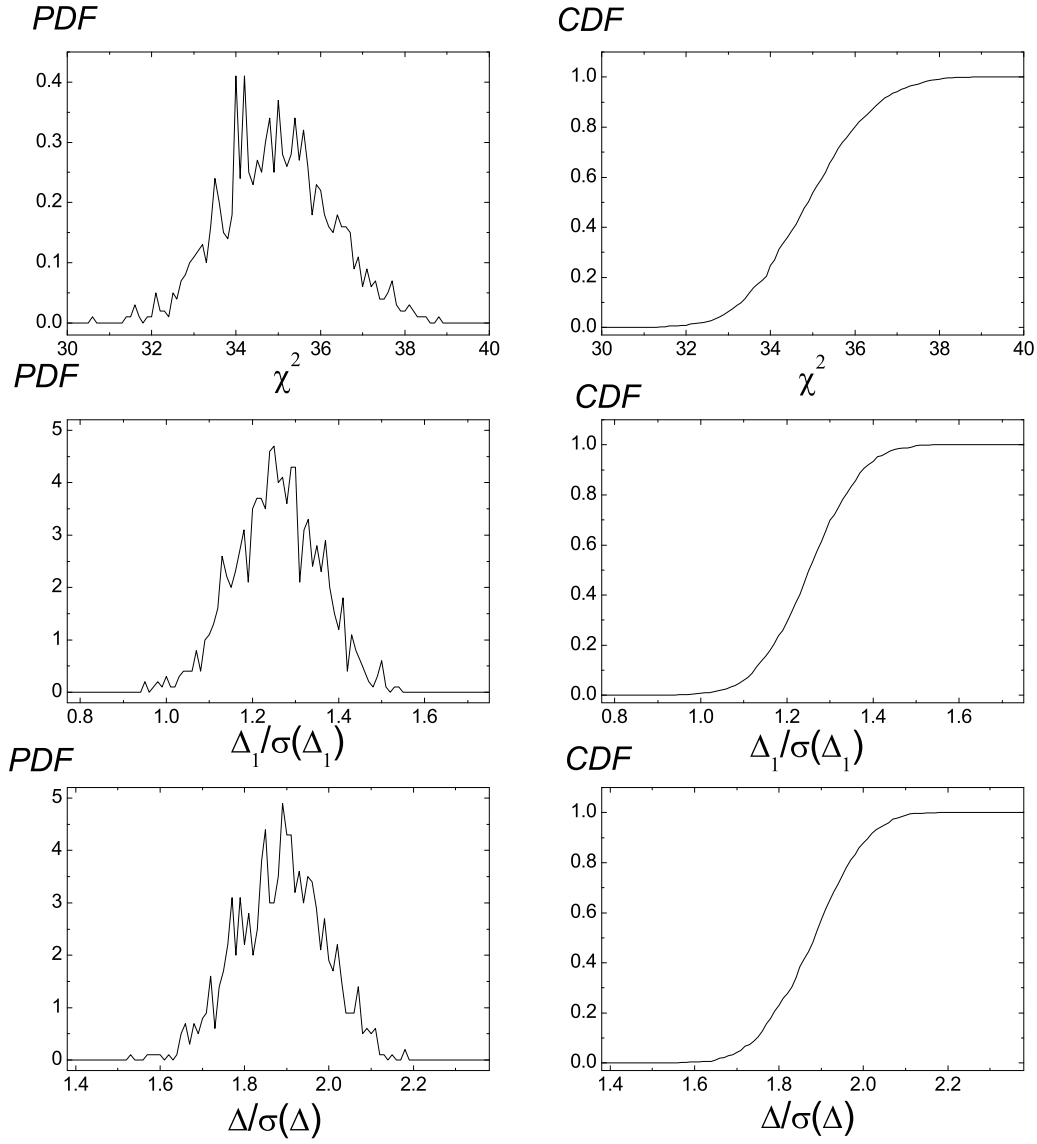


Figure 1: Probability density function (PDF, left panel) and cumulative distribution function (CDF, right panel) for statistics χ^2 , $\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$.

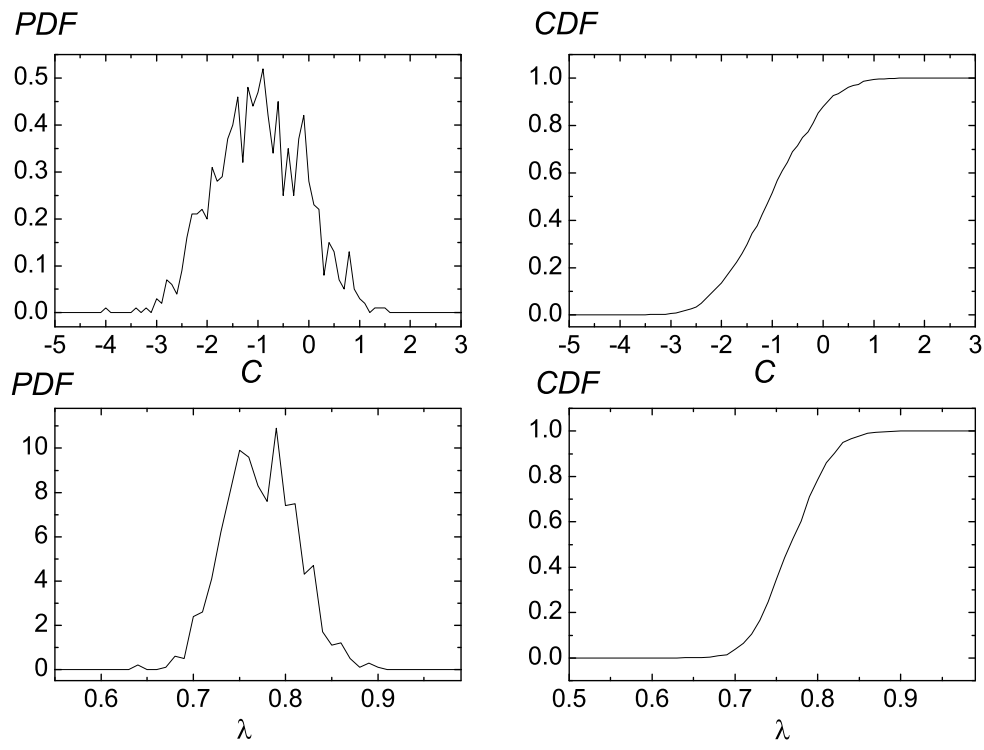


Figure 2: Probability density function (PDF, left panel) and cumulative distribution function (CDF, right panel) for statistics C and λ .

Table 1: The frequency of anisotropy of very rich clusters located in superclusters.

Multiplicity	The angle P	The angle δ_D	The angle η
N=4	0.84	0.74	0.84
N=5-7	0.31	0.90	0.79
N=8-10	0.43	0.57	0.43

orientation of the galaxy plane; namely the position angle of the great axis of the galaxy image P and the angles describing the orientation of the normal to the galaxy plane: δ_D and η . The polar angle δ_D is the angle between the normal to the galaxy plane and the main plane of the coordinate system; the azimuthal one η is angle between the projection of this normal onto the main plane and the direction toward the zero initial meridian and positional angle (see for example Flin & Godłowski 1986, Paper I). In the present paper, as well as in Paper II, we analyzed the sample of 43 very rich clusters (having 100 and even more members) belonging to superclusters.

The existence of an alignment for each particular cluster belonging to our sample was analyzed in Paper I (Table 4). On that basis it was possible to analyze the frequency of alignments in our sample of galaxy clusters attributed to superclusters (Table 1). To first order, the data indicate that anisotropy decreases with supercluster multiplicity.

It is also possible to analyze the alignment of clusters belonging to superclusters in more detail. The standard method of approach for galactic alignments is an analysis of the distribution of the angles, which provides information connected with the orientation of galaxies. That approach was proposed by Hawley & Peebles (1975), who analyzed the distribution of position angles using χ^2 testing, Fourier testing, and first autocorrelation testing. One should note that there were several modifications and improvements to the original Hawley & Peebles (1975) method (Flin & Godłowski 1986, Kindl 1987, Godłowski 1993, 1994, Aryal & Saurer 2000, Godłowski et al., 2010). Godłowski (2012) made a significant improvement to the original Hawley & Peebles (1975) method and showed its usefulness in the analysis of galaxy orientations in clusters. In Godłowski (2012) the mean values of the analyzed statistics were computed. The null hypothesis H_0 assumed that the mean value of the analyzed statistics was that expected for the case of a random distribution of analyzed angles. The results were compared with theoretical predictions as well as with the results obtained from numerical simulations.

Following the Godłowski (2012) method, we analyzed our sample of 43 clusters belonging to superclusters. In Paper II we analyzed only the χ^2 statistic and

statistics obtained on the basis of Fourier testing. χ^2 statistics was studied as

$$\chi^2 = \sum_{k=1}^n \frac{(N_k - N_{0,k})^2}{N_{0,k}} \quad (1)$$

where N_k is the number of galaxies within k -th angular bin and as $N_{0,k}$ is the expected number of galaxies per bin, n is the number of bins.

If the theoretical probability function PF is uniform, then $N_{0,k}$ are equal.

In all applied statistical tests, the entire range of the investigated θ angle (as θ we accept $\delta_D + \pi/2$, η or P).

If deviation from isotropy is a slowly varying function of the angle, one can use the Fourier test (Hawley & Peebles, 1975):

$$N_k = N_{0,k}(1 + \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k) + \dots \quad (2)$$

If the theoretical probability function PF is symmetric with respect to the value $\theta = \pi/2$, we obtain the following expressions for the Fourier coefficients:

$$\Delta_{11} = \frac{\sum_{k=1}^n (N_k - N_{0,k}) \cos 2\theta_k}{\sum_{k=1}^n N_{0,k} \cos^2 2\theta_k} \quad (3)$$

$$\Delta_{21} = \frac{\sum_{k=1}^n (N_k - N_{0,k}) \sin 2\theta_k}{\sum_{k=1}^n N_{0,k} \sin^2 2\theta_k} \quad (4)$$

The probability function has amplitude

$$\Delta_1 = (\Delta_{11}^2 + \Delta_{21}^2)^{1/2} \quad (5)$$

with the standard deviation of the amplitude

$$\sigma(\Delta_1) \approx \left(\frac{2}{nN_0}\right)^{1/2}. \quad (6)$$

The amplitude Δ was calculated using higher Fourier coefficients till 4θ , according Godłowski (1994).

Here we extend our analysis in comparison with Paper II using autocorrelation and Kolmogorov - Smirnov (K-S) testing. Autocorrelation test we applied in form:

$$C = \sum_{k=1}^n \frac{(N_k - N_{0,k}) \cdot (N_{k+1} - N_{0,k+1})}{\sqrt{N_{0,k} N_{0,k+1}}} \quad (7)$$

Because of the small number of galaxies in some clusters, we made 1000 simulations of the distribution of position angles in 43 fictitious clusters, each cluster with the number of galaxy members identical to the real cluster. On this basis we obtained the probability density function (PDF) and the cumulative distribution function (CDF) seen in Fig. 1 and Fig. 2. The expected value for the analyzed statistics and its variance were computed as well. In Table 2 we present

the average values of the analyzed statistics, the corresponding standard deviations, the standard deviations in the sample, and the standard deviations for the distribution of P angles. Details of the applied statistics were presented in previous papers (Paper I and Godłowski, 2012).

It is now possible to compare the results obtained for the actual sample of 43 clusters with that obtained from numerical simulations (right hand side of Table 2). If we assume that the true distribution of position angles is uniform, then an exact value for the probability that the analyzed statistic included a specific chosen value can be obtained from CDF (Figs. 1 and 2).

However, one should note that our procedure computes the mean values of the analyzed statistics. When the errors are normally distributed (Gaussian), which is the case at least for some statistics analyzed in Godłowski (2012), the parameters are estimated by the maximum-likelihood method. The distribution should have an asymptotic normal (Gaussian) appearance, which was checked by Godłowski (2012) with the use of the Kolmogorov - Lilliefors test (Lilliefors, 1967). There it was shown that a Gaussian approximation works well, which made the interpretation of the results much easier. For the sample of all 43 clusters located in superclusters the distribution of position angles of galaxy members in the cluster is anisotropic and the departure from isotropy is usually greater than 3σ (see Table 2), with the exception of the first autocorrelation test where the effect is less than 2σ . For the angles which gave the spatial orientation of galaxy planes (δ_D and η angles) the anisotropy is even greater than in the case of position angles P . In our opinion that can be attributed to incorrectly assumed shapes for the galaxies. That problem was analyzed in detail by Godłowski & Ostrowski (1999) and Godłowski (2011). Those studies were based on Tully's NGC Catalogue (1988). In that catalogue, while calculating galaxy inclination angles, Tully assumed that the "true" ratio of axes for galaxies is 0.2, which, as we have shown in the above papers, is a rather poor approximation, especially for non-spiral galaxies (Godłowski, 2011). For that reason, the previous study concentrated on the analysis of position angles. In our present analysis, presented below, the effect is not especially important because, for the case of analyzing the spatial orientation of galaxy planes, our interest is only to show how the alignment changes with membership of the clusters in a supercluster as well as with supercluster multiplicity.

The main goal of this study is connected with finding trends appearing in the data. In Paper I, while analyzing entire samples of 247 rich Abell clusters, we found that the alignment increases with cluster richness. In the analyzed sample of 43 clusters of galaxies belonging to superclusters we do not observe that effect (Table 3). This conclusion is significantly different from the

result obtained in Paper I for the whole sample of 247 rich Abell clusters. We suppose that such a difference can be traced to environmental effects during the formation of superclusters. Note that the distributions of analyzed angles are anisotropic in both cases: for the entire collection of 247 rich Abell clusters and for the subsample of 43 clusters belonging to superclusters.

In Paper II we presented an analysis of 43 clusters (Table 4.), binned according to supercluster multiplicity. One can observe that the anisotropies seem to decrease with supercluster richness. For that reason, in the present paper we decided to perform an unbinned analysis of the linear regression between values of the analyzed statistics and supercluster multiplicity. The results are presented in the Table 5. We analyzed statistics $T = \frac{a}{\sigma(a)}$, the Students' T distribution with $n - 2$ degrees of freedom. For $n = 46$ at the significance level $\alpha = 0.05$, the critical value $T_{\text{crit}} = 1.68$. We tested the H_0 hypothesis that the value of the analyzed statistic does not depend on supercluster richness against the H_1 hypothesis that it decreases with supercluster richness. From Table 5 we can conclude that only in the case of the η angle, the anisotropy decreases with the supercluster richness and is statistically significant on a significance level of 0.05.

4. Conclusions

In the present paper we investigated a sample of 43 rich Abell galaxy cluster belonging to a supercluster and containing at least 100 members in the considered area. We found that the orientation of galaxies in the analyzed cluster was not random. However, in contrast with the results of Godłowski et al. (2010), we detect that for our sample the alignment of galaxies does not depend on cluster richness. The differences between samples analyzed in these studies are as follows. In Godłowski et al. (2010), we analyzed a sample of 243 rich Abell galaxy clusters, while in the present paper we analyzed only subsamples of galaxies belonging to supercluster. Nevertheless, for both samples we observed that the distributions of analyzed angles P , δ_D and η , which specify the orientation of galaxies in space, are not random. We also found that the alignment decreases with supercluster richness, although the effect is statistically significant only for azimuthal angles (η angles). The results obtained, which include the dependence of galaxy alignment on cluster location inside or outside a supercluster as well as supercluster multiplicity, clearly support the influence of environmental effects on the origin of galaxy angular momenta. The problem of obtaining the angular momenta of galaxies in a structure is rather complicated since several mechanisms play roles. According to the major scenarios for galaxy formation, in some cases the angular momentum of galaxies results from local

anisotropic collapse of protostructures, in others due from a tidal torque mechanism. Moreover, clusters can merge, introducing additional factors which influence the observed distribution of galaxy angular momenta. This suggests that environment played a crucial role in the origin of galaxy angular momentum. In a very simple and naive picture, if the alignment of galaxies is primordial, the strongest effect should be observed in small structures. In the present paper we analyzed only the sample very rich clusters. For final confirmation or rejection of this hypothesis, it is necessary to enlarge the analysis taking into account a sample of poorer clusters. Fortunately, our basic PF Catalogue (Panko & Flin, 2006) will allow us to perform such an analysis in the future.

Acknowledgements. This research has made use of the NASA Astrophysics Data System, and was partially supported by grant BS052 of Jan Kochanowski University (Kielce, Poland).

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Table 2: The theoretical (random) and observational values of statistics.

Test	Simulations				Observations	
					PA	
	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$	\bar{x}	$\sigma(x)$
χ^2	34.9592	1.2843	0.0406	0.0287	38.772	1.574
$\Delta_1/\sigma(\Delta_1)$	1.2567	0.0983	0.0031	0.0022	1.797	0.148
$\Delta/\sigma(\Delta)$	1.8846	0.1027	0.0032	0.0023	2.339	0.148
C	-0.9750	0.8593	0.0274	0.0192	0.611	1.120
λ	0.7729	0.0392	0.0012	0.0009	0.932	0.057

Table 3: The results of the linear regression analysis: value of the analyzed statistics as a function of the cluster richness for clusters belonging to superclusters.

	χ^2	$\Delta_1/\sigma(\Delta_1)$	$\Delta/\sigma(\Delta)$	C	λ
angle	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$
P	0.028 ± 0.039	-0.0004 ± 0.0037	-0.0029 ± 0.0037	0.027 ± 0.028	0.0004 ± 0.0014
δ	0.026 ± 0.037	-0.0008 ± 0.0023	0.0020 ± 0.0024	0.031 ± 0.032	0.0004 ± 0.0011
η	0.125 ± 0.040	0.0060 ± 0.0030	0.0082 ± 0.0028	0.112 ± 0.041	0.0018 ± 0.0009

Table 4: The statistical analysis: value of the analyzed statistics for different supercluster multiplicities.

angle	Test	$N = 4$	$N = 5 - 7$	$N = 8 - 10$
P	χ^2	43.30 ± 2.42	34.99 ± 2.11	36.65 ± 1.55
	$\Delta_1/\sigma(\Delta_1)$	1.99 ± 0.25	1.50 ± 0.16	1.89 ± 0.32
	$\Delta/\sigma(\Delta)$	2.57 ± 0.23	2.08 ± 0.18	2.42 ± 0.36
	C	2.53 ± 1.60	-0.99 ± 1.27	-0.37 ± 4.05
	λ	1.01 ± 0.08	0.81 ± 0.06	1.01 ± 0.21
δ_D	χ^2	54.52 ± 10.69	48.68 ± 4.57	89.07 ± 18.56
	$ \Delta_{11}/\sigma(\Delta_{11}) $	3.13 ± 0.63	2.76 ± 0.57	6.41 ± 0.84
	$\Delta_c/\sigma(\Delta_c)$	4.76 ± 0.72	4.40 ± 0.34	6.97 ± 1.09
	C_c	29.73 ± 9.48	22.53 ± 4.46	51.59 ± 16.36
	λ_c	2.22 ± 0.33	2.01 ± 0.13	3.39 ± 0.55
η	χ^2	83.60 ± 12.09	95.57 ± 9.74	43.01 ± 5.44
	$\Delta_1/\sigma(\Delta_1)$	5.43 ± 0.86	5.83 ± 0.71	2.02 ± 0.29
	$\Delta/\sigma(\Delta)$	6.30 ± 0.88	7.31 ± 0.66	3.19 ± 0.38
	C	49.31 ± 12.34	54.25 ± 10.50	5.26 ± 2.93
	λ	2.24 ± 0.28	2.37 ± 0.19	1.07 ± 0.13

Table 5: The results of linear regression analysis: value of the analyzed statistics as a function of supercluster multiplicity.

	χ^2	$\Delta_1/\sigma(\Delta_1)$	$\Delta/\sigma(\Delta)$	C	λ
angle	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$	$a \pm \sigma(a)$
P	-1.093 ± 0.772	0.018 ± 0.074	0.001 ± 0.074	-0.249 ± 0.563	0.014 ± 0.029
δ	6.839 ± 2.937	0.527 ± 0.181	0.435 ± 0.195	4.282 ± 2.651	0.206 ± 0.126
η	-7.863 ± 3.553	-0.574 ± 0.255	-0.558 ± 0.256	-7.772 ± 3.668	-0.178 ± 0.078